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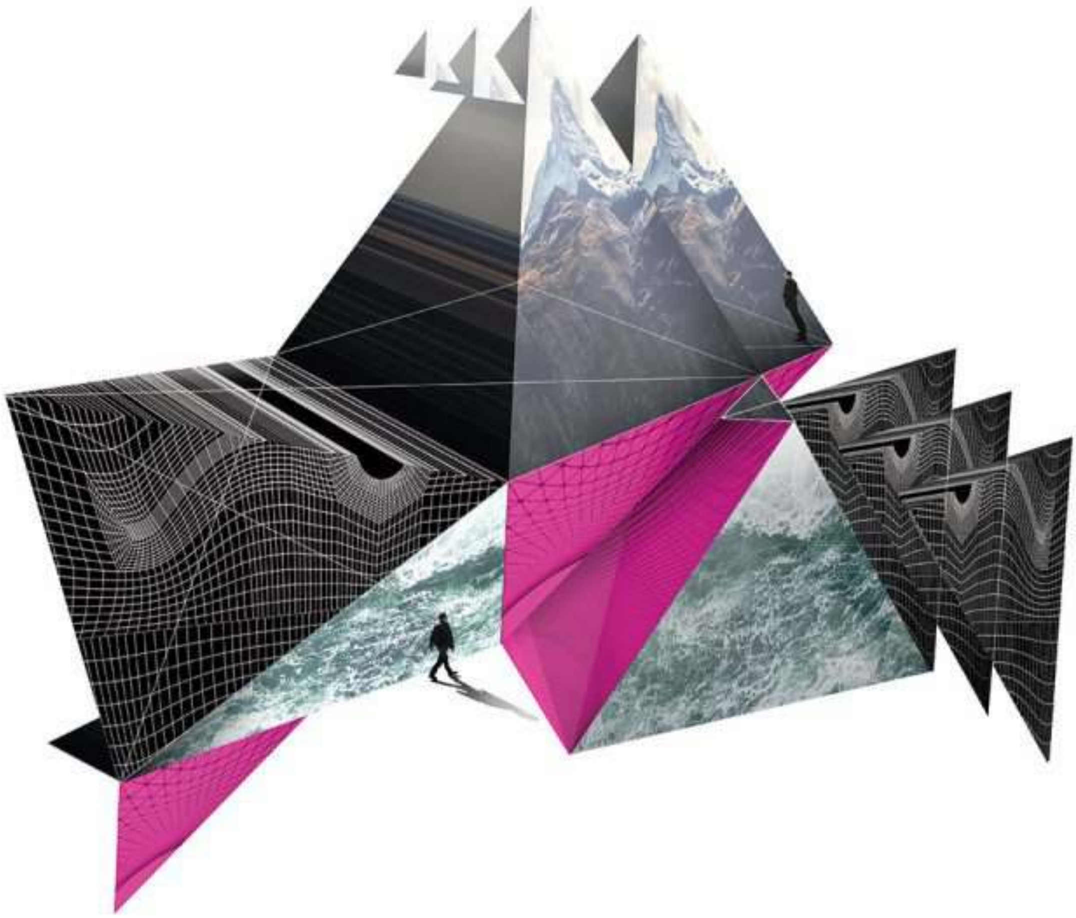
The geometry that could reveal the true nature of space-time

The discovery of an exquisite geometric structure is forcing a radical rethink of reality, and could clear the way to a quantum theory of gravity



Skizzomat

By **Anil Ananthaswamy**

FOR years after the physicist Richard Feynman died, his 1970s yellow-and-tan Dodge minivan lay rusting in a garage near Pasadena, California. When it was restored in 2012, special effort was made to repaint the giant doodles that adorned its bodywork. They don't look like much – simple combinations of straight lines, loops and squiggles. But it is no exaggeration to say these Feynman diagrams revolutionised particle physics. Without them, we might never have built the standard model of particles and forces, or discovered the Higgs boson.

Now we could be on the cusp of a second, even more far-reaching transformation. Because even as Feynman's revolution seems to be fizzling out, physicists are discovering hints of deeper geometric truths. If glimpses of exquisite mathematical structures that exist in dimensions beyond the familiar few can be substantiated, they would seem to point the way to a better understanding not just of how particles interact, but of the nature of reality itself.

It was a hard road that led to the standard model, this monumental theoretical construct that describes all the particles of the quantum world and the forces that act on them, except for gravity. The starting point came in the 1930s and early 1940s, when physicists investigating quantum electrodynamics, the theory of how charged particles and electromagnetic fields interact, embarked on calculations of "scattering amplitudes" – the probabilities of different outcomes in a given particle interaction. But the calculations proved maddeningly difficult. For a while they seemed impossible.

Then along came Feynman. In 1949, he showed an intuitive way to tackle the calculations, using doodles that could literally be drawn on cocktail napkins. Take, for example, the interaction of two electrons. The electrons are depicted by two straight lines that are approaching each other. But before the lines meet, the electrons interact by exchanging a "virtual" photon, drawn as a squiggly line, causing the two straight lines to move apart. The two electrons have repelled each other.

This is the simplest and most likely interaction. But for a full picture, you have to come up with all possible Feynman diagrams a given interaction could have, capturing all the different ways in which the particles can influence each other. One of the electrons might emit and absorb a virtual photon, for instance, creating a squiggly loop, which can interact with itself to generate more loops. The basic procedure is that you turn each possible diagram into an algebraic formula and work them all out to get the scattering amplitude.

The more virtual particles, the more complicated the calculations. So why invoke virtual particles at all? It does seem strange given that they are not real particles. A real particle is essentially a consistent ripple in an energy field, one that persists over time. But when real particles interact, they can cause temporary ripples in underlying quantum fields, such as an electromagnetic field. These are called virtual particles, and they are used in Feynman diagrams for several reasons.

The first is that dealing with them rather than with fields makes the maths more tractable. The other great advantage is that they help physicists visualise everything as the well-defined interactions between point-like particles, as opposed to the hazy goings-on between particles and fields. This fits nicely with the intuitive principle of locality, which holds that only things in the same spot in space and time can interact. Finally, the technique also helps enforce the principle of unitarity, which says that the probability of all outcomes should add up to 1.

Sticky like gluons

Feynman diagrams worked beautifully when applied to photons and electrons, and became a staple of physics, being used to predict the outcome of experiments to astonishing precision. But once physicists started to tackle quantum chromodynamics, the theory of interactions involving quarks and gluons – the basic components of the protons and neutrons at the heart of atoms – things got sticky. There were so many virtual particles, and ways each interaction could happen, that every calculation using Feynman diagrams required "heroic efforts of computation", says Jacob Bourjaily at the University of Copenhagen's Niels Bohr Institute in Denmark.

This much became obvious in the 1980s, when the US was building the ill-fated Superconducting Super Collider in Texas. It was going to smash protons into each other, so it was imperative to understand the interaction of gluons, which hold together the quarks that make up protons. But it seemed impossible. "Their complexity is such that they may not be evaluated in the foreseeable future," one group of physicists wrote at the time.

Then there was an unexpected turn of events. In 1986, Stephen Parke and Tomasz Taylor from Fermilab near Batavia, Illinois, used Feynman diagrams and supercomputers to calculate the likelihoods of different outcomes for interactions involving a total of six gluons. A few months later, they made an educated guess at a one-line formula to calculate the same thing. It was spot on. More than 200 Feynman diagrams and many pages of algebra had been reduced to one equation, and the researchers had no idea why.

What was clear was that virtual particles were a big part of the problem. "Every single Feynman diagram is a fantasy," says Bourjaily. A fantasy in the sense that we have no way of observing the virtual particles they depict. What we do know is that the wild proliferation of mathematics required to account for them are very real, resulting in ridiculously unwieldy calculations.

Almost 20 years passed before another clue arrived. In 2005, Ruth Britto, Freddy Cachazo, Bo Feng and Edward Witten were able to calculate scattering amplitudes without recourse to a single virtual particle and derived the equation Parke and Taylor had intuited for that six-gluon interaction.

This time there was a lead on what the BCFW method might mean. It was inspired by a view of space-time called twistor theory, which had been developed in the late 1960s and early 1970s by Roger Penrose at the University of Oxford. The primary objects of this theory are not particles, but rays of light, or twistors. "You can think of the universe as built up out of these rays, and points of space and time emerge at the places where these rays meet," says Andrew Hodges, one of Penrose's colleagues at Oxford.

Hodges showed that the various terms used in the BCFW method could be interpreted as the volumes of tetrahedrons in twistor space, and that summing them up led to the volume of a polyhedron. The trouble was that his insight only worked for the simplest, most likely interaction of gluons with specific properties. For more complicated particle interactions, the resultant geometric objects were utterly bewildering. Their connection with real particle dynamics was intriguing, but the maths was too difficult.

It took Nima Arkani-Hamed and his team at the Institute of Advanced Studies (IAS) in New Jersey, including his then students Jaroslav Trnka and Bourjaily, to join the dots. Building on the seemingly esoteric work of pure mathematicians, the team arrived at a mind-boggling conclusion: the scattering amplitude calculated with the BCFW technique corresponds beautifully to the volume of a new mathematical object. They gave a name to this multi-dimensional concatenation of polyhedrons: the amplituhedron.

Skizzomat

It's best to think of the amplituhedron not as a real object but as an abstraction. It's a mathematical structure that gives us an elegant way to encode the calculations that tell us how likely a particle interaction is to play out in a certain way. The details of the interaction, meaning the number and properties of the particles involved and the forces involved, dictate the dimensions and facets of the corresponding amplituhedron – and that contains the answer. So there are actually many amplituhedra, one for each possible way in which a set of particles may interact.

The contrast with Feynman diagrams is stark. On one hand you may have to draw a thousand diagrams and use supercomputers, on the other you can get the same answer by calculating the volume of a single geometric object, even if the maths involved is far from trivial. "It translates the physics problem into a purely mathematical problem – calculate the volume of that object," says Trnka, who is now at the University of California, Davis.

It may transform physics, too – potentially nudging the door ajar to a unified theory of everything. That's because the amplituhedron does not embody unitarity and locality, those core principles baked into reality as described by Feynman diagrams. Scattering amplitudes that obey the laws of locality and unitarity do emerge from amplituhedra. But unlike in Feynman diagrams, the amplituhedron does not start with space-time that has these properties. "The thing that you calculate will be unitary and local," says Trnka. "It's a consequence of the geometry."

If so, locality is not a fundamental feature of space-time but an emergent one. That amounts to a radical rethink of reality (see "It sounds crazy, but…"), and one that could finally help us with a solution to one of the biggest questions in physics: how gravity behaves at the very smallest scales.

Locality and gravity don't sit well together. In order to precisely determine what happens at a given point in space-time, you have to zoom in closer and closer and examine smaller and smaller intervals of time. Quantum mechanics says that as one gets increasingly precise, the energy fluctuations in that region of space-time become bigger. Now, energy is mass, and mass has gravity, so incredibly high amounts of mass in a very tiny region of space ends up forming a black hole, which makes it impossible to see what's going on – and dashes any hopes of insight about the quantum nature of gravity. So, if gravity and quantum mechanics have to coexist, locality has to go.

The amplituhedron suggests that it can, potentially clearing the way for a quantum theory of gravity. That would finally let us understand what really goes on inside black holes and maybe even at the moment of the big bang – secrets of the universe that are theoretically impenetrable today.

"Ultimately, space-time and quantum mechanics might emerge as one"

If Arkani-Hamed is correct, that might just be the start. "If we are going to lose something as dramatic as the idea of space-time, it's very unlikely that it leaves any of physics unaffected," he told the audience at the String-Math 2016 conference in Paris, France. "It must show up everywhere. It must show up even in situations where we think we understand things perfectly well."

Naturally, there is a catch. Over the past few years, Arkani-Hamed and his colleagues have demonstrated that the amplituhedron works for a "toy" model of particle interactions that involves supersymmetry, a theory in which all standard model particles have partner particles. But the standard model, our best description of reality, is not supersymmetric.

If that sounds like a killer blow, it isn't. "The toy model is closer to reality than any other toy that people have played with over the last three decades," said Arkani-Hamed in a talk at the IAS in April this year. Indeed, for some of the simplest, most likely particle interactions, the calculations using the amplituhedron agree with results obtained using standard calculation methods. Crucially, the new method holds for all four-dimensional theories of massless particles, supersymmetric or otherwise. The standard model has its origins in this class of theories, so it's entirely plausible that it will work there too. "This correspondence with geometry is a general thing," says Bourjaily. "It's a statement about four-dimensional theories."

Now the challenge is to extend this geometric way of thinking to more realistic models of particle interactions, and ultimately include gravity by doing away with locality. It's not going to be that simple, though. Which might be why Witten, who is also at the IAS, is simultaneously impressed and circumspect. “Perhaps [the amplituhedron] is the closest we have to a unified picture, at least of some of the questions,” he says. “There have been so many surprises in the study of these scattering amplitudes that it is rather hard to speculate on future directions. But it is pretty clear that there is a lot still to discover.”

Arkani-Hamed is confident that, ultimately, we will see that space-time and quantum mechanics emerge as one. “In this baby example that’s exactly what happens,” he said in Paris. “There is no way in this geometry to decouple the piece which is space-time from the piece which is quantum mechanics. It’s all one and the same aspect of the underlying positive geometry.”

It sounds crazy, but…

History shows that radical new ways of thinking about reality are well worth grappling with. Take Newton’s laws of motion. Given the position of a particle and all the forces acting on it, you can show deterministically – by describing cause and effect – how it goes from point A to point B. But there is another way to think about the particle’s path. It’s called the principle of least action. It says that a particle will take the path that minimises a quantity called classical action, which is the average value of the particle’s kinetic energy minus its potential energy along the path.

This principle felt weird to minds trained in classical physics. “[No one] thought that particles smelled around all possible paths and took the one that minimised this silly number,” says Jacob Bourjaily of the Niels Bohr Institute in Copenhagen, Denmark. “It’s a very weird starting point for classical physics.” What’s more, the theory appears non-deterministic because a particle’s trajectory isn’t obvious at the onset. Nonetheless, the principle of least action makes the same predictions as Newton’s laws, suggesting that determinism is emergent, and the calculations involved are easier.

Significantly, this way of thinking about reality was more in tune with the emerging quantum mechanics, and led to things like Feynman diagrams, which opened the door to the subatomic world. The hope now is that a strange mathematical structure called an amplituhedron, which does the same things as Feynman diagrams but in a counterintuitive way, may lead physicists towards a greater prize.

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